

# XXX Fund

## Weekly Risk Management Report

Based on prime broker statements, dated 22/03/2005

Daniel HERLEMONT, YATS Finances & Technologies  
<http://www.yats.com>

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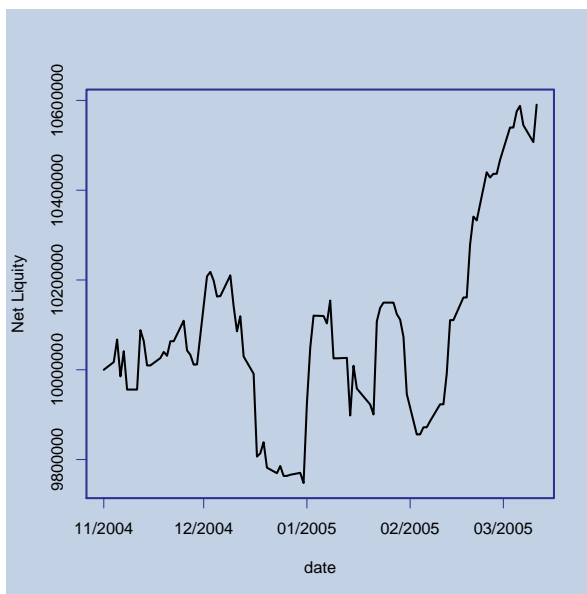
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## 1 Summary

Mark to Market	
Net Liquidity value	10590497 euros
Last day variation of the Net Liquidity	83307 euros
Total P&L from 12/11/2004	590497 euros
	5.9%
no open position	
Annualized Performance	17.85%
Risk Measures	
Annualized Volatility of the Fund	10.5%
VaR (one month, at 95% Cornish Fisher)	-3.86%
Daily Var Limit	-92441 euros
Maximum Drawdown	4.6%
Current drawdown	0%
Risk Adjusted Performance Measures	
Sharpe ratio	1.71
Morningstar Risk Adjusted Return (MRAR)	15.97%
Stutzer Index	1.59
Higher Moments	
Skewness	0.432
Kurtosis (in excess)	1.53

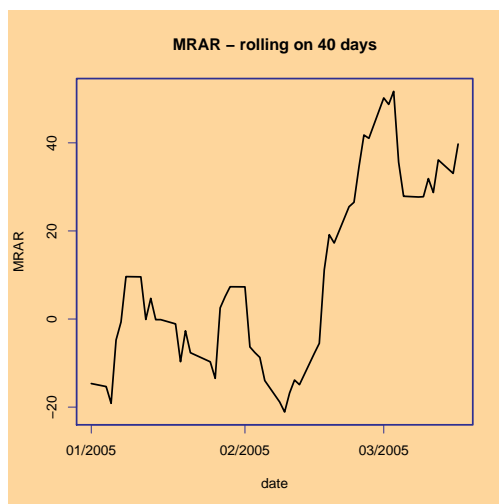
Notes on Risk Adjusted Performance Measures:

- Risk adjusted measures (Sharpe, MRAR, Stutzer index) are computed without considering risk free rate
- The Morningstar measure, also known as the Stutzer index (see annex ??), can be interpreted very easily from an investor perspective: this is the certainty equivalence for a typical investor endowed with a risk aversion about 3. That is, a typical investor should prefer to invest in the fund rather than investing in a risk free asset with return less than 16.0% per year. Note that the MRAR of the underlying BUND future is 2.02% per year.



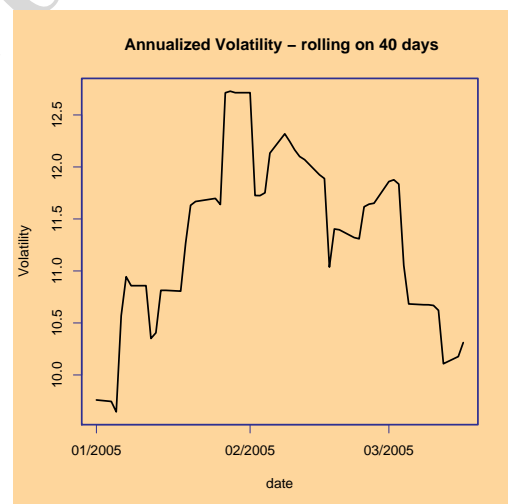
The last 20 days mark to market Net Liquidity values:

Date	Net Liquidity	P&L
22/02/2005	9923089	0
23/02/2005	9991523	68435
24/02/2005	10110537	119014
25/02/2005	10110537	0
28/02/2005	10160970	50433
01/03/2005	10160970	0
02/03/2005	10278762	117792
03/03/2005	10341244	62482
04/03/2005	10332640	-8604
07/03/2005	10439881	107241
08/03/2005	10428275	-11605
09/03/2005	10436502	8226
10/03/2005	10436502	0
11/03/2005	10467323	30821
14/03/2005	10539594	72271
15/03/2005	10539594	0
16/03/2005	10575534	35940
17/03/2005	10587928	12394
18/03/2005	10544894	-43033
21/03/2005	10507190	-37704
22/03/2005	10590497	83307



Last value of the rolling MRAR is 39.7%. The 95% confidence interval (bootstrap method) of the MRAR is:

2.5% 97.5%  
-16.7 63.4



Last value of the rolling Volatility is 10.3%

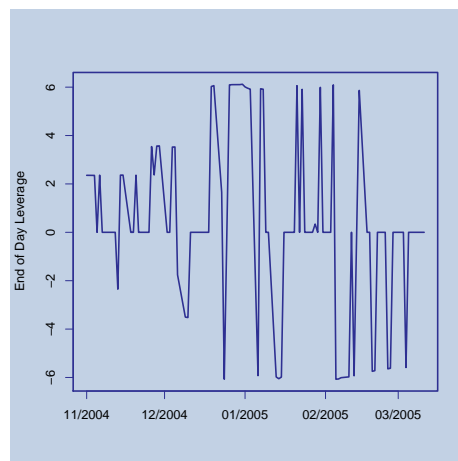
## 2 Leverage

### 2.1 Funds End of Day Leverage

The leverage is defined

$$\frac{\theta_t * S_t}{NetLiquidity_t}$$

where  $\theta_t$  is the position held in contracts (positive if long, negative if short) and  $S_t$  is the notional price, i.e. the price of one contract times 10 euros per basis point, in other word the quoted price shall be multiplied by 1000.



### 2.2 Optimal leverage

The growth rate of the Fund (or the MRAR) is not a linear function of the leverage. Excessive leverage is the main cause of financial disasters (see for example, Thorp[5], Ziemba [6]).

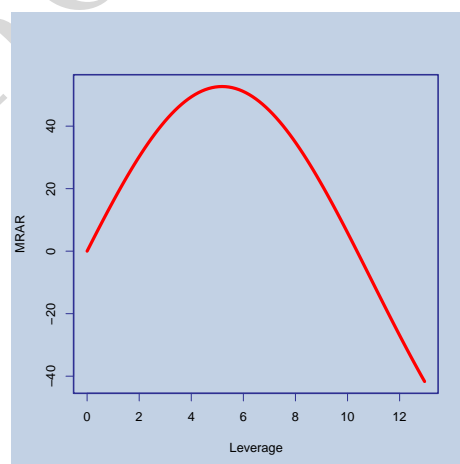
We can determine the optimal leverage and sensitivity of the Fund performance to mis adjusting the leverage.

**Betting two times the optimal leverage will probably lead to ruin !!!!**

The following figure represents the MRAR (see 10.3, for definition) as a function of the leverage. This is the leverage of an hypothetical investor who decide to invest in the Fund and made optimal choice to maximize its MRAR.

This figure shows that the annualized MRAR is highly sensitive to the actual leverage.

The MRAR is similar to a growth rate, the actual growth rate is the MRAR for the logarithm utility function with risk aversion 1 which is much more aggressive than the standard MRAR with a power utility function and risk aversion 3 (i.e  $\gamma = 2$  in MRAR definition).



The optimal leverage is  $w^* = 5.18$ , with maximum  $MRAR^* = 52.7\%$  per year.

In practice, this optimal leverage is very close to

$$\frac{1}{\gamma} \frac{\mu}{\sigma^2} = 5.01$$

with  $\gamma = 3$ , the relative risk aversion,  $\mu$  the mean return and  $\sigma$  the volatility of the Fund.

Leveraging two times the optimal leverage yields  $MRAR(10.4) = -0.0761\%$  per year only. It is much safer to trade at optimal leverage divided by 2:  $MRAR(2.59) = 37.4\%$  or even 4:  $MRAR(1.30) = 20.4\%$

### 3 Maximum Drawdown

#### 3.1 Current Maximum Drawdown

Maximum Drawdown is defined as the maximum of the running maximum drop (see annex 10.2 for a more precise definition).

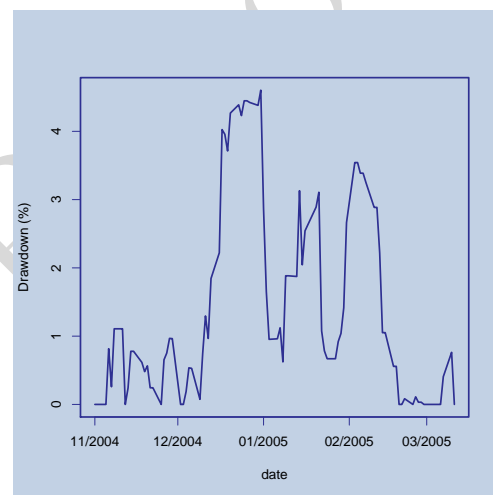
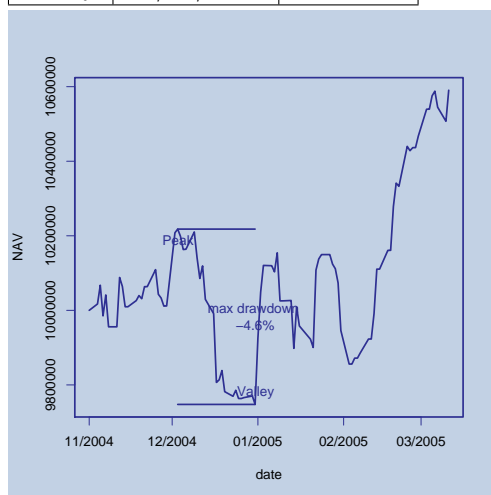
Objective is to keep the maximum drawdown below 30%.

The current maximum drawdown is 4.6%

This maximum drawdown represents the losses from the following peak to valley:

	Date	NAV
Peak	14/12/2004	10217944
Valley	11/01/2005	9747683

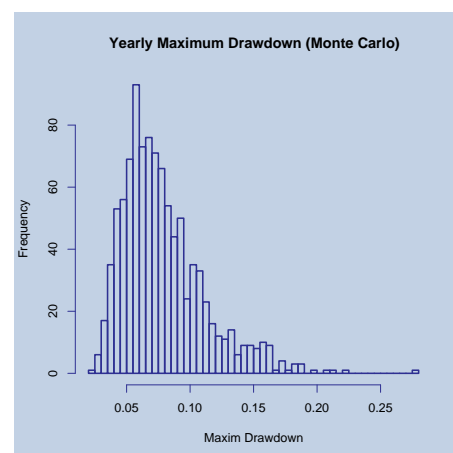
The current drawdown is 0%.



#### 3.2 Simulated Maximum Drawdown

Monte Carlo Simulation for the Maximum Drawdown over one year:

- Mean yearly simulated MDD: 7.84%
- standard deviation of simulated MMD: 3.26%
- the 95% confidence interval of MDD is [3.52%, 16.0%]



## 4 Value At Risk

### 4.1 VaR and Stop Loss Limits

The Value at Risk objective is defined to be 4% of the fund value for a one month horizon and 95% confidence level. To backtest the actual VaR against this VaR objective, we transform the one month VaR to a daily VaR objective using the squared time rule. Considering there are 21 trading days per month, the daily VaR objective can be defined from the monthly VaR objective as follows:

$$VaR(1day; 95\%) = \frac{1}{\sqrt{21}} VaR(21days; 95\%)$$

With  $VaR(21days; 95\%) = -4\%$  objective, the daily VaR objective is

$$VaR(1day; 95\%) = -0.894\%$$

Hence, with a current value of 10590497.4498498 euros, the daily VaR objective in euros is

$$DailyVarLimit = 0.00894 \times 10590497 = -92441$$

Stop loss shall be set so that the daily loss cannot exceed the "Daily VaR Limit" -92441 euros.

With a position of  $N$  contracts, stop loss shall be set at

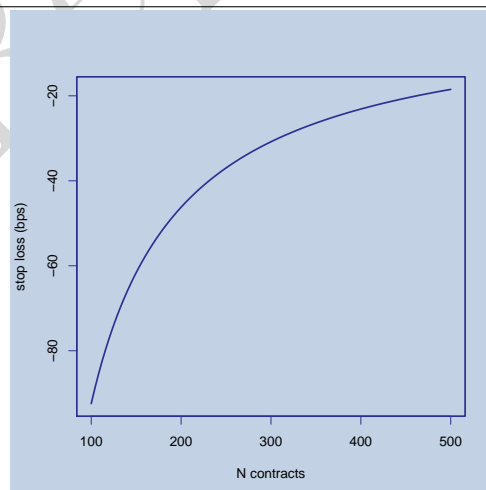
$$S = P + \frac{VaR_{limit}}{N * 1000}$$

where  $S$  is the stop loss,  $P$  is the the price that has been used to compute the VaR limit (i.e close price or settlement price),  $VaR_{limit}$  is the Daily Var Limit and  $N$  the number of contracts.

For example, with  $N = 500$  contracts long, and  $VaR_{limit} = -92441$  euros VaR limits, stop loss should be set at

$$\begin{aligned} StopLoss(500 \text{ contracts}) &= \frac{100 \times 92441}{500 \times 1000} \\ &= 18 \text{ bps} \end{aligned}$$

As the number of contracts increases, the stop loss size becomes smaller.

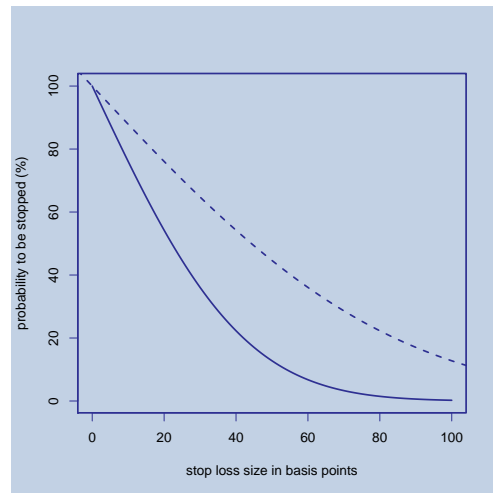


This figure represents the number the mandatory and minimal stop loss size as a function of the number of contracts. Note that the daily VaR limit is negative. If long (short),  $N$  is positive and stop loss is below (above) the settlement price

As the number of contracts increases, the stop loss size becomes smaller and the probability to be stopped increases. This probability to be stopped during a trading day can be roughly estimated (see annex 10.1), it mainly depends on the volatility of daily increments that is 32.8 basis points

stop loss trail in basis points	probability to be stopped
5	88%
10	76%
20	54%
30	36%
40	22%
50	13%
100	0.23%

Stop loss are essential for Risk Management. However, they are far from perfect. Stop loss execution are usually very costly in terms of micro structure market conditions (bid/ask spread, spikes, order book depth, ...). Stop loss policy is consistent with trends. If not (in case of mean reversion for example), stop loss results in much lower returns.



The probability to be stopped also increases with the volatility. The dashed curve represents the probability in case of doubling volatility

Conversely, starting from the probability to be stopped, we can determine the number of contracts. Let  $p$  be this probability, then the stop loss barrier  $S - P = m = \sigma \Phi^{-1}(1 - p/2)$  and the number of contracts  $N = V/(m * 1000)$ . For example, with a probability  $p = 0.2$ , the stop loss shall be set at  $S - P = m = -42$  bps and the number of contracts is  $N = 220$

## 4.2 VaR estimates

### 4.2.1 Historical VaR

The historical daily VaR can be read from quantiles of daily profit and losses in euros:

0%	1%	5%	10%	50%	100%
-184456	-134386	-89755	-66098	0	207527

For example, the historical daily VaR at 95% confidence level is the 5% quantile, i.e.: -89755.1597193656 euros. The 0% and 100% quantile are the min and the max of the daily P&L.

Applying the time square root rule, the monthly VaR is estimated to be the daily VaR times  $\sqrt{21}$ , that is -411309.8134268, or  $\boxed{-3.88\%}$  of the Net Liquidity.

### 4.2.2 Normal VaR

This VaR estimate is based on the assumption of a normal distribution of rate of return of the Net Liquidity, with daily mean  $\mu_1 = 0.0652\%$  and daily standard deviation  $\sigma_1 = 0.659\%$

The daily Value at Risk at 95% confidence is

$$VaR(1day) = \mu_1 - 1.645\sigma_1 = -1.02\%$$

The one month estimated VaR is based on the assumption that the return are identically and independently normally distributed:

$$\mu_{21} = 21\mu_1 = 1.37\%$$

and standard deviation

$$\sigma_{21} = \sqrt{21}\sigma_1 = 3.02\%$$

Hence, one month VaR is

$$VaR(1month) = \mu_{21} - 1.645\sigma_{21}$$

Based on current Net Liquidity rate of returns and volatility and the normal hypothesis, the one month VaR at 95% confidence level is  $\boxed{-3.595\%}$  (objective is  $-4\%$ )

#### 4.2.3 Cornish Fisher VaR

Under the normal hypothesis both skewness and excess kurtosis should be equals to zero. In fact, as shown in section 6.2, the distribution of funds returns exhibit both a significant skewness and kurtosis in excess. One can use a Taylor expansion of the normal quantile to take into account the skewness and kurtosis:

$$z \approx z_0 + \frac{1}{6}(z_0^2 - 1)S + \frac{1}{24}(z_0^3 - 3z_0)K - \frac{1}{36}(2z_0^3 - 5z_0)S^2$$

where  $z_0$  is the  $1 - \alpha$  quantile of the normal distribution:  $N(z_0) = 1 - \alpha$ ,  $S$  is the skewness and  $K$  the kurtosis in excess.

With a skewness  $S = 0.432$  and kurtosis in excess  $K = 1.53$ , the Cornish Fisher correction is

$$z = 1.73$$

rather than,  $z_0 = 1.64$

The Corsnish Fisher one month VaR is:

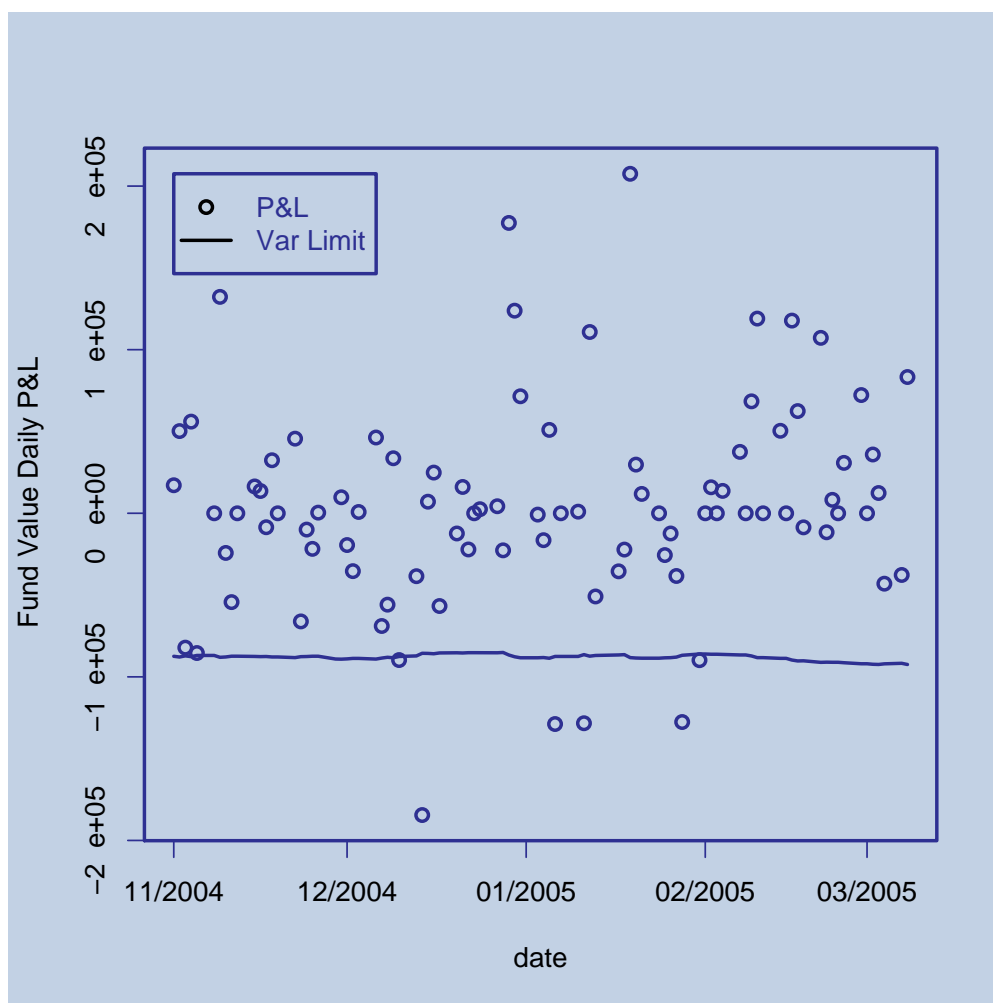
$$\boxed{VaR(CornishFisher; 1month) = \mu_{21} - 1.73\sigma_{21} = -3.86\%}$$

(objective is  $-4\%$ )

### 4.3 Backtesting the Value At Risk

The most straightforward way to backtest a VaR model is to plot daily P&L against predicted VaR, as recommended by RiskMetrics [2] [3].

The following figure represents the daily P&L with the daily VaR objective.



Usually, we are (back)testing the 95% VaR on year lag, i.e. 252 trading days. The number of expected exceptions is  $12.5 = 0.05 * 252$

Currently, the number of available data is 91 days, then the number of expected exceptions is  $91 * 0.05 = 4.55$ . This is to be compared with the actual number of exceptions observed: 6.

To accept or reject the 95% VaR (hypothesis  $H_0$ ), we need to perform a statistical test to know if the difference between expected and actual is significant or not? Over a time period of  $T$  trading days, the number of exceptions  $N$  is distributed according to a binomial distribution:  $Prob(exception = N) = \binom{T}{N} p^N (1-p)^{T-N}$ . To accept or reject this null hypothesis, a simple binomial test has to be performed:

- actual estimate of the confidence level of the VaR is 0.934, rather than 0.95 for a VaR at 95% confidence.
- The p value<sup>1</sup> of the test is 0.5,
- the 95 percent confidence interval for VaR confidence is [0.862, 0.975]. This interval shall contain 0.95, that is the tested null hypothesis.

In conclusion, the 95% VaR objective can be accepted

<sup>1</sup>the p value represents the probability of making an error when we reject the null hypothesis and the null hypothesis is true. In other words we cannot reject the null hypothesis at any level smaller than the p value

## 5 The cost of dynamic trading

- Total number of contracts traded: 83860
- Number of trading days: 92
- Average number of contracts traded / day = 912
- Daily turn over 861%

In dynamic trading, trading cost may be very important. In addition to the broker's transaction cost (2 euros per contract), the largest part of the cost in future market comes from the market frictions (bid/ask spread, market depth, ...). Hopefully, the BUND future market is very liquid and we can take a minimalist hypothesis that consist in considering this friction cost as a tick value, that is 1 basis point or 10 euros per contract. Hence, the minimal total cost of trading one BUND contract is 12 euros. However, even considering this minimalist assumption, the cost of dynamic trading seems very high. With a total of 83860 traded contracts from the starting date (12/11/2004), the cost of dynamic trading is about  $83860 * 12 = 1006320$  euros, that is 10938 euros per trading day, representing a loss of -0.103% per trading day or -26.0% annualized. Saying in other words, without frictions, the NetLiquidity would be 11596817.4498498 euros (16.0% from start), rather than 10590497.4498498 euros (5.9% from start).

In case of pre defined entry or exit points, it is much more preferable to use limit orders rather than market orders.

## 6 Other Statistical Tests and Indicators

### 6.1 t-stat

Student t-statistics of the fund value rate of return is: 0.944, the p value is 0.347, the 95% confidence interval of the annualized return is: [-18.1%, 51%]

The student t-test is to test the significance of return with respect to a null hypothesis of zero return. The t statistic is computed as  $t = \sqrt{n}\mu/\sigma$  with  $n$  the number of trading days,  $\mu$  the daily mean return, and  $\sigma$  the daily volatility.  $t$  is distributed as a student distribution with degree of freedom  $n$ . Usually, a t stat of 2 is considered as excellent. When  $n = 252$ , i.e. the t stat coincide with the annualized sharpe ratio  $t\text{-stat} = \sqrt{252}\frac{\mu_{1d}}{\sigma_{1d}} = \mu_{1y}/\sigma_{1y} = \text{sharpe}$

### 6.2 Normality testing of the Net Liquidity returns

Normality testing is useful to asses the validity of normality assumption in VaR estimates.

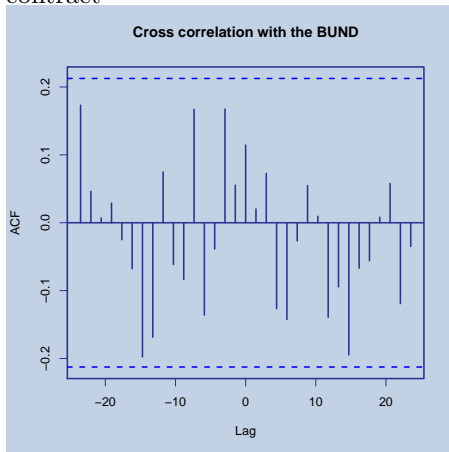
skewness	kurtosis
0.432	1.53

	Jarque Bera	Shapiro Wilk
statistic	13	0.946
p value	0.001	0.001

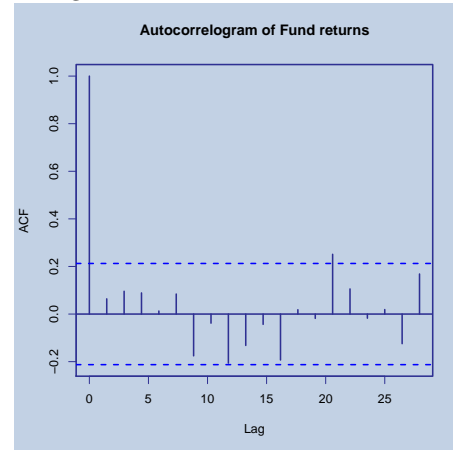
## 7 Correllation studies

Correlation of daily returns with the BUND returns is 0.114
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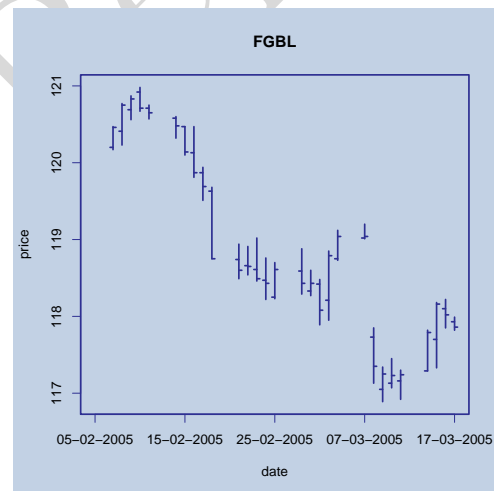
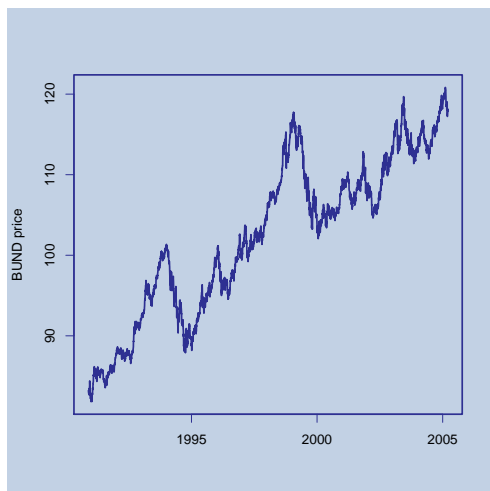
The cross correlation at different lags show a significant correlation with the underlying BUND contract



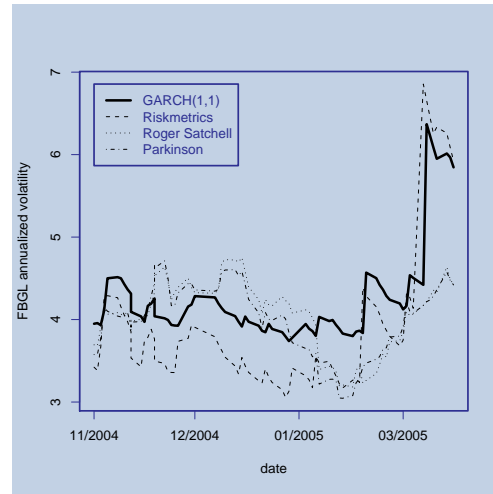
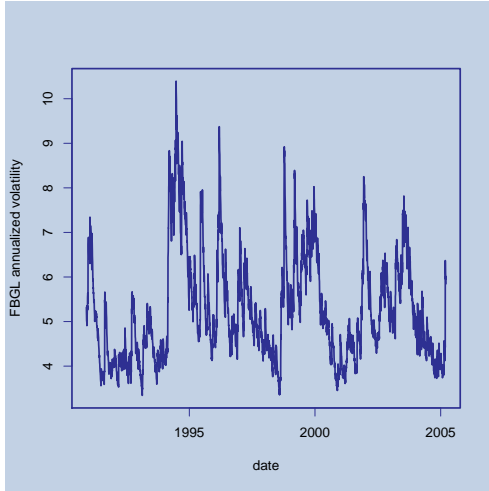
The autocorrelation of daily Fund returns display no significant autocorrelation.



## 8 The BUND contract



The current volatility is historically low.  
According to volatility model, the annualized volatility is 4.42%



GARCH	5.85%
Riskmetrics	5.92%
Rogers Satchell	4.42%
Parkinson	4.42%

For the FBCL contract, the GARCH(1,1) has the following parameters:

$$a_0 = 1.39e - 07 \quad a_1 = 0.0422 \quad b_1 = 0.946$$

The unconditional variance is

$$E[\sigma_t^2] = \frac{a_0}{1 - (a_1 + b_1)}$$

i.e, we recover the unconditional volatility  $\sqrt{E[\sigma_t^2]} * 252 = 5.42\%$ .

The parameter  $\lambda = a_1 + b_1$  can be interpreted as the mean reverting parameter.  $1/(1 - \lambda)$  is the mean time to recover a previous level after a shock.

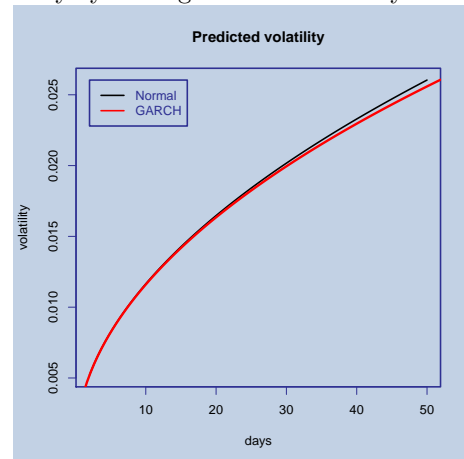
The mean reverting factor is  $\lambda = 0.988$ , hence the volatility cycle length is about 84 days.

For a GARCH(1,1) model, the volatility prediction T days ahead is the following:

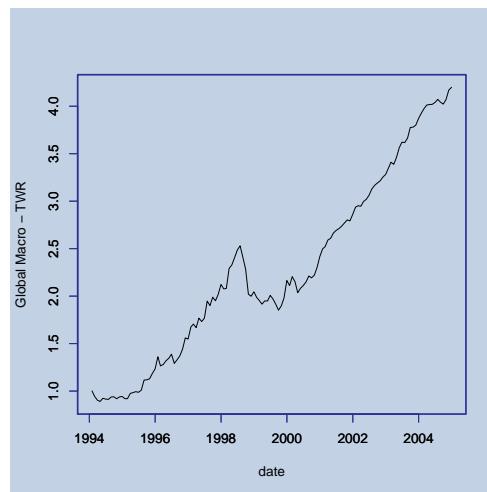
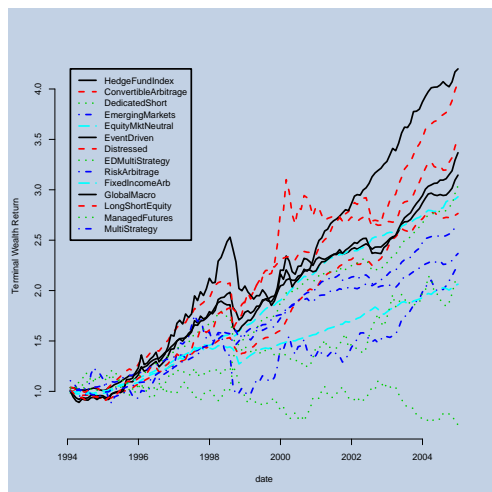
$$\sigma_t^2(T) = T\sigma^2 + (\sigma_t^2 - \sigma^2) \frac{1 - (a_1 + b_1)^T}{1 - (a_1 + b_1)}$$

where  $\sigma^2$  is the unconditional variance:

$$\sigma^2 = \frac{a_0}{1 - (a_1 + b_1)}$$



## 9 CSFB/Tremont Hedge Funds indices



Return of CSFB Hedge Funds indices over the last 12 months:

Strategy	01 2004	02 2004	03 2004	04 2004	05 2004	06 2004	07 2004	08 2004	09 2004	10 2004	11 2004	12 2004
HedgeFundIndex	1.70	1.40	0.28	-0.58	-0.23	0.34	-0.31	0.14	1.01	1.28	2.65	1.61
ConvertibleArbitrage	1.42	0.29	0.42	0.46	-1.33	-0.76	-0.20	0.28	-0.07	-0.29	1.08	0.69
DedicatedShort	-1.73	0.34	-2.56	4.23	0.76	-1.25	8.12	1.27	-1.91	-1.78	-7.71	-4.87
EmergingMarkets	2.53	1.39	1.83	-3.31	-1.81	0.87	-0.14	1.83	2.33	2.40	2.69	1.39
EquityMktNeutral	0.82	0.79	-0.11	-0.34	0.21	0.84	0.31	2.13	0.54	0.03	0.26	0.86
EventDriven	2.16	0.95	0.45	0.51	0.09	0.96	0.00	0.45	1.27	1.24	3.27	2.28
Distressed	2.42	0.89	0.59	0.66	0.26	1.06	0.52	0.56	1.23	1.86	2.67	1.92
EDMultiStrategy	2.01	1.05	0.29	0.47	-0.09	0.93	-0.34	0.38	1.35	0.73	3.88	2.62
RiskArbitrage	0.83	0.50	0.73	-0.58	0.44	0.25	-1.52	0.18	0.63	0.92	1.67	1.32
FixedIncomeArb	1.23	0.87	-0.49	1.34	0.63	0.71	0.70	-0.41	-0.78	1.14	1.12	0.61
GlobalMacro	1.45	1.19	0.97	0.14	0.05	0.48	0.82	-0.75	-0.49	1.22	2.42	0.72
LongShortEquity	2.00	1.75	0.20	-1.40	-0.36	0.66	-1.42	0.09	2.36	1.44	3.24	2.57
ManagedFutures	1.09	6.89	-0.86	-6.46	-1.05	-2.84	-1.95	-1.53	1.96	4.82	5.83	0.72
MultiStrategy	1.60	0.39	0.41	0.32	-0.13	0.09	-0.29	0.41	0.57	0.59	2.11	1.25

Indicators of CSFB Hedge Funds indices:

Strategy	$R$	$\sigma$	TWR	Sh	MDD	S	K	MRAR	Stutzer
HedgeFundIndex	11.34	8.15	3.15	0.90	13.81	0.10	1.85	10.26	1.32
ConvertibleArbitrage	9.80	4.66	2.76	1.24	12.04	-1.42	3.53	9.45	1.79
DedicatedShort	-2.12	17.66	0.67	-0.35	46.55	0.89	1.98	-6.38	0.12
EmergingMarkets	9.16	17.04	2.23	0.30	45.15	-0.60	3.90	4.30	0.51
EquityMktNeutral	10.32	3.02	2.93	2.09	3.55	0.28	0.18	10.17	3.36
EventDriven	11.86	5.84	3.37	1.35	16.04	-3.39	22.63	11.26	1.52
Distressed	13.85	6.71	4.06	1.47	14.32	-2.77	16.46	13.05	1.57
EDMultiStrategy	10.85	6.19	3.04	1.11	18.54	-2.58	16.31	10.19	1.39
RiskArbitrage	8.25	4.34	2.37	0.98	7.60	-1.28	5.92	7.95	1.63
FixedIncomeArb	6.88	3.83	2.06	0.75	12.47	-3.17	16.05	6.64	1.43
GlobalMacro	14.68	11.59	4.20	0.92	26.78	0.00	2.17	12.44	1.18
LongShortEquity	12.72	10.60	3.51	0.82	15.05	0.22	3.41	10.89	1.14
ManagedFutures	7.76	12.20	2.10	0.31	17.74	0.03	0.33	5.42	0.62
MultiStrategy	9.41	4.35	2.66	1.24	7.11	-1.24	3.40	9.10	1.86

Notations:  $\bar{R}$  is the annualized return,  $\sigma$  is the annualized volatility,  $TWR$  is the terminal wealth return,  $Sh$  is the Sharpe ratio,  $MDD$  is the maximum drawdown,  $S$  is the skewness,  $K$  is the kurtosis;  $MRAR$  is the

Morningstar Risk Adjuster Return (see 10.3)

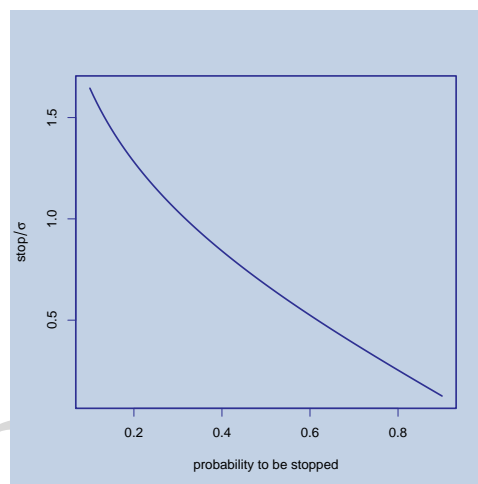
## 10 Annex

### 10.1 Probability to be sopped

The probability to be stopped is the same as the probability that the maximum (or minimum) of the increment will be above the stop loss. Assuming that the daily increments follow a random walk without drift, this probability is

$$Prob(maximum(t) > a) = 2 \left( 1 - \Phi \left( \frac{a}{\sigma\sqrt{t}} \right) \right)$$

with  $\Phi$  the normal distribution function. Those probabilities are certainly under estimated, since daily increments have fatter tails than a normal distribution and much more spikes than brownian motion



### 10.2 Maximum Drawdown

Maximum Drawdown is defined as the maximum of the running maximum drop. Let  $W_t$  the wealth at time  $t$ , and  $M_t$  the running maximum on  $0, t$

$$M_t = \max_{0 \leq s \leq t} W_s$$

The running drawdown  $D_t$  is the running loss since the highest high

$$D_t = \frac{M_t - W_t}{M_t}$$

note that  $D_t \geq 0$  <sup>2</sup>

The maximum drawdown  $MDD_t$  at time  $t$  is the running maximum of the drawdown

$$MDD_t = \max_{0 \leq s \leq t} D_s$$

### 10.3 Morningstar Risk Adjusted Return

The Morningstar Risk-Adjusted Return (also known as the Stutzer index) is defined as the certainty equivalence for a power utility function  $U(W) = -W^{-\gamma}/\gamma$ , for  $\gamma > -1$  and  $\gamma \neq 0$  or  $U(W) = \ln(W)$  for  $\gamma = 0$  (hence relative risk aversion is  $\gamma+1$ ). Constant relative risk aversion also implies that the investor's beginning of period wealth has no effect on the ranking of portfolios. Instead of holding a risky portfolio, the investor could buy a risk-free asset. Let  $R_b$  be the return on the risk-free asset. In comparing risky portfolios to the risk-free asset with  $R_b$  returns, we assume that the investor initially has all wealth invested in the risk-free asset. Then,

$$U = \frac{(1 + R_g)^{-\gamma}}{\gamma}$$

<sup>2</sup>an other popular and equivalent measure is the ratio  $\frac{M_t}{W_t}$

where  $R_g$  is the geometric excess return:

$$R_g = \frac{1 + R}{1 + R_b} - 1$$

The certainty equivalent geometric excess return of a risky investment is the guaranteed geometric excess return that the investor would accept as a substitute for the uncertain geometric excess return of that investment. Letting  $R_g^{CE}(\gamma)$  denote the certainty equivalent geometric excess return for a given value of  $\gamma$ , this means that:

$$U(1 + R_g^{CE}(\gamma)) = E[U(1 + R_g)]$$

$$R_g^{CE}(\gamma) = \begin{cases} (E[(1 + R_g)^{-\gamma}])^{-1/\gamma} - 1 & \gamma > -1 \quad \gamma \neq 0 \\ e^{E[\ln(1 + R_g)]} - 1 & \gamma = 0 \end{cases}$$

$MMAR(\gamma)$  is defined as the annualized value of  $R_g^{CE}(\gamma)$  using the time series average of  $(1 + R_g)^{-\gamma}$ . For example, if  $R_i$  are the monthly rate of returns, then, for  $\gamma \neq 0$

$$MRAR(\gamma) = \left( \frac{1}{n} \sum_{i=1, n} \frac{1 + R_i}{1 + R_b} \right)^{-12/\gamma} - 1$$

for  $\gamma = 0$

$$MRAR(0) = \left( \sqrt[n]{\prod_{i=1, n} \frac{1 + R_i}{1 + R_b}} \right)^{12} - 1$$

12 shall be replaced by 252 for daily returns.

## 10.4 Stutzer Index

The main concern for investors is the probability of under performing a benchmark on average. The Stutzer index [4] rewards those portfolios that have a lower likelihood of underperforming a specified benchmark on average. This measure penalizes negative skewness and high kurtosis (for given levels of mean returns and variance).

Denote a portfolio p's excess (i.e. net of benchmark) rate of return by  $R_{pt}$ , and denote the time average over T:

$$\bar{R}_{pt} = \frac{1}{T} \sum_{t=1}^T R_{pt}$$

Now assuming the portfolio has a positive expected return, the law of large number implies that  $Prob(\bar{R}_{pt}) \rightarrow 0$  as  $T \rightarrow 0$ . In iid return process (and a wide variety of other processes), this probability will converge to zero at exponential rate  $I_p$ :

$$Prob(\bar{R}_{pt} \leq 0) = \frac{c}{\sqrt{T}} e^{-I_p T}$$

for large  $T$ , where  $c$  is a constant that depends on the distribution... A fund manager that is averse of receiving a non positive return over the benchmark will choose portfolio at the maximum possible rate  $I_m$

Cramer's theorem can be used to compute the rate  $I_p$ :

$$I_p = \max_{\theta} -\log E(e^{\theta R_p})$$

where  $\theta$  is a number less than zero. When the returns are normally distributed,

$$I_p = \frac{1}{2} \lambda_p^2$$

that is half square the Sharpe ratio.

Finding the portfolio that maximizes the Stutzer consist in solving

$$I_m = \max_{w_1, \dots, w_n} \max_{\theta} - \log \sum_{i=1, T} e^{\theta \sum_{i=1}^n w_i (R_{it} - R_{0t}) + R_{0t}}$$

The Stutzer Index was formerly used for the Morningstar, Inc. International Global Star Ratings of mutual funds.

The Stutzer index shares some nice properties with the Sharpe ratio for normal distributions [1], the Stutzer being more general since those properties hold for any distribution:

- if  $X$  and  $Y$  are independent then

$$\text{Stutzer}(\alpha X + \beta Y) = \text{Stutzer}(X) + \text{Stutzer}(Y)$$

- the maximum is achieved for  $\alpha = \lambda(X)$  and  $\beta = \lambda(Y)$ , where  $\lambda(Z)$  depends only on  $Z$

That is, two independent assets (or strategies) have their Stutzer indexes added when they are put together in the right proportions.

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